Calculation of effective collecting area of lobster eye with respect to full point spread function

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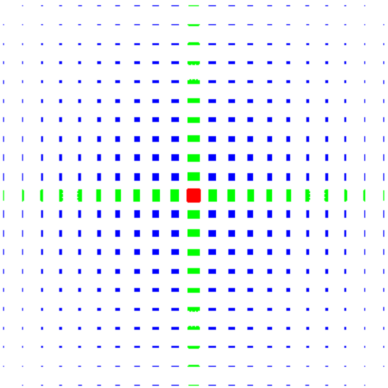
Typical form of image acquired by lobster eye

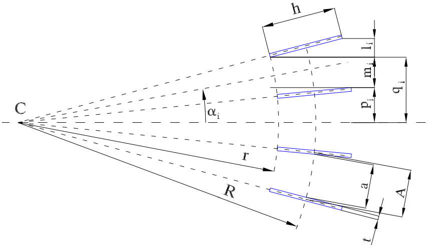
Colors are not related to intensity but to the origin of the part of the image.



Focal spot is formed by rays which are reflected in vertical and horizontal direction.
Arms of cross are formed by rays which are reflected either in vertical or in horizontal direction.
Direct rays, i.e. rays are not reflected at all form spots around the cross.
Reflections of higher order are not taken into account.
Questions: How many photos come to each of parts of the image? How large is the corresponding collecting area of the optics?

Idea - look at cross-section

The image represents cross-section of one stack of Schmidt lobster eye or cross-section of chamber of Angel lobster eye.



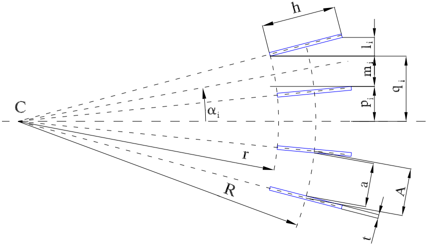
We know, how to calculate $L = \sum l_i$ which characterizes amount of reflected rays [1]

$$L = 2R\eta \frac{\widetilde{\mathcal{R}}(2\zeta) - 2\widetilde{\mathcal{R}}(\zeta) + \widetilde{R}(0)}{\zeta}; \quad \widetilde{\mathcal{R}}(\theta) = \int\int \mathcal{R}(\theta)\mathrm{d}\theta^2; \zeta = \frac{a}{h}, \eta = \frac{a}{a+t} \quad (1)$$

$\widetilde{\mathcal{R}}(\theta)$ represents coefficient of reflectivity of the mirror at grazing angle θ.
Let us calculate the length $M = \sum m_i$ that corresponds to amount of rays coming through the stack of Schmidt system directly or through the chamber of Angel system not being reflected in specified direction.

[1] Exp. Astron. 47 (2019) 161-175

Derivation of equation for M



$$p_i = \left(R + \frac{h}{2}\right)\left(|\alpha_i| - \frac{a}{2R}\right) \quad q_i = \left(R - \frac{h}{2}\right)\left(|\alpha_i| + \frac{a}{2R}\right) \tag{2}$$

$$m_i = q_i - p_i = a - h|\alpha_i| \tag{3}$$

$$M = \sum_{\alpha_i \in [-\zeta, \zeta]} m_i = \sum_{\alpha_i} a - h|\alpha_i| \approx \int_{-\zeta}^{\zeta} \frac{a - h|\alpha|}{\frac{A}{R}} \mathrm{d}\alpha = 2\frac{R}{A}\int_0^{\zeta} a - h\alpha \mathrm{d}\alpha \tag{4}$$

$$M = R\eta\zeta \tag{5}$$

Assumed: $\sin(\alpha) \approx \alpha$, A/R is small

Application to 2D optics and corresponding areas

For Schmidt optics, M and L differ for individual stacks. Let it be denoted L_1 and M_1 are related to the first stack and L_2, M_2 are related to the second stack.
Collecting area related to everything which appears on the image equals

$$S_{TOTAL} = (L_1 + M_1)(L_2 + M_2) = \underbrace{L_1 L_2}_{S_F} + \underbrace{L_1 M_2 + L_2 M_1}_{S_A} + \underbrace{M_1 M_2}_{S_D} \quad (6)$$

Angel optics use square chambers and therefore M, L does not differ in vertical and horizontal direction. Therefore

$$S_{TOTAL} = (L + M)^2 = \underbrace{L^2}_{S_F} + \underbrace{2LM}_{S_A} + \underbrace{M^2}_{S_D} \quad (7)$$

- S_F = input area from which all photons come to focal spot
- S_A = input area from which all photons come to arms of the cross
- S_D = input area from which all photons come through the optics directly (not being reflected)

Calculations are shown in specific cases. For simplicity, Angel lobster eye is assumed.

Ideal mirror

$$\mathcal{R}(\theta) = 1 \quad \forall\theta \geq 0 \tag{8}$$

$$L = 2R\eta\zeta = 2M \tag{9}$$

$$S_F = L^2 = 4R^2\eta^2\zeta^2 = \frac{4}{9}S_{TOTAL} \tag{10}$$

$$S_A = 2LM = 4R^2\eta^2\zeta^2 = \frac{4}{9}S_{TOTAL} \tag{11}$$

$$S_D = M^2 = R^2\eta^2\zeta^2 = \frac{1}{9}S_{TOTAL} \tag{12}$$

We see $S_F = S_A$, i.e. the same amount of photons form focal spot and arms of the cross. For real mirror which does not have 100% reflectivity, $L < 2M$ and therefore $S_F < S_A$.

Ideal mirror

Numerical test with Angel lobster eye of parameters $R = 500$mm, $A = 1.11172$mm, $t = 0.1$mm, $h = 24$mm.
$L = 38.36$mm
$M = 19.18$mm

	Calculation by presented model	Ray-tracing simulation
S_F $\left[\text{mm}^2\right]$	1472	1462
S_A $\left[\text{mm}^2\right]$	1472	1464
S_D $\left[\text{mm}^2\right]$	368	367
S_{TOTAL} $\left[\text{mm}^2\right]$	3311	3293

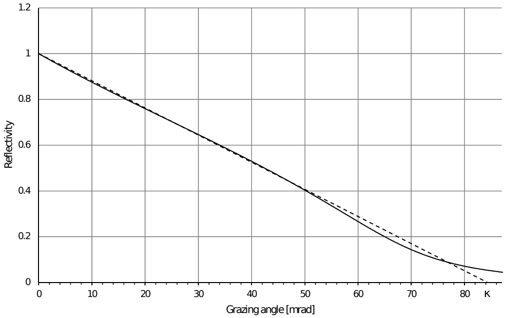
Ray-tracing simulations were made by LOPSIMUL software *www.lopsimul.eu* using simplified ray-tracing algorithm published in Exp. Astron. (2016) 41:377-392; DOI 10.1007/s10686-016-9493-2.

Linear model of reflectivity

Here, the reflectivity is approximated by the function

$$\mathcal{R}(\beta) = \begin{cases} 1 - \frac{\zeta}{\kappa} & \forall 0 \leq \zeta \leq \kappa \\ 0 & \forall \beta \geq \kappa \end{cases} \tag{13}$$

Example: Thick gold layer and unpolarized X-rays at 1keV photon energy, $\kappa = 84.31$mrad. Reflectivity data were acquired from *http://henke.lbl.gov/optical_constants/*. Solid line represents real reflectivity data. Dashed line represents the approximation.



Linear model of reflectivity

Assume $\zeta_{\mathrm{optimal}} = \kappa/2$ that represents the case when S_F is maximized.

$$
\begin{aligned}
L &= R\eta\frac{\kappa}{2} && (14)\\
M &= R\eta\frac{\kappa}{2} && (15)
\end{aligned}
$$

$$
\begin{aligned}
S_F &= L^2 = R^2\frac{\kappa^2}{4} = \frac{1}{4}S_{TOTAL} && (16)\\
S_A &= 2LM = R^2\frac{\kappa^2}{2} = \frac{1}{2}S_{TOTAL} && (17)\\
S_D &= M^2 = R^2\frac{\kappa^2}{4} = \frac{1}{4}S_{TOTAL} && (18)
\end{aligned}
$$

We see $2S_F = S_A$, i.e. photons forming arms of the cross are twice more than photons forming focal spot.

Linear model of reflectivity

Numerical test with Angel lobster eye of parameters $R = 500$mm, $A = 1.11172$mm, $t = 0.1$mm, $h = 24$mm. A is chosen to maximize S_F.
$L = M = 19.18$mm

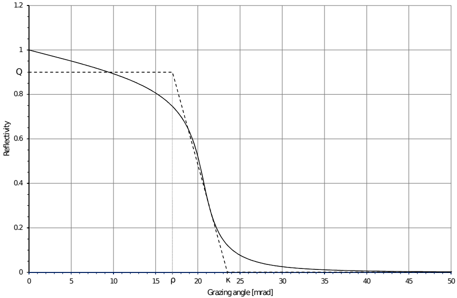
	Calculation by presented model	Ray-tracing simulation
S_F $\left[\mathrm{mm}^2\right]$	368	367
S_A $\left[\mathrm{mm}^2\right]$	736	734
S_D $\left[\mathrm{mm}^2\right]$	368	367
S_{TOTAL} $\left[\mathrm{mm}^2\right]$	1472	1467

Smoothed step model

Other model of wide usage is

$$\mathcal{R}(\theta) = \begin{cases} Q & \forall 0 \leq \theta \leq \rho \\ Q\frac{\kappa-\theta}{\kappa-\rho} & \forall \rho \leq \theta \leq \kappa \\ 0 & \forall \theta \geq \kappa \end{cases} \quad (19)$$

Here, θ is the grazing angle and ρ and κ are constants fulfilling $0 < \rho < \kappa$.



Example: Titanium at 2keV unpolarized photon energy, $Q = 0.9, \rho = 17\text{mrad}, \kappa = 23.5\text{mrad}$.

Smoothed step model

Equations for L are complex. They can be found in *Exp. Astron. 47 (2019) 161-175*.
We show example with Angel lobster eye of parameters $R = 500\text{mm}$, $A = 0.44513\text{mm}$, $t = 0.1\text{mm}$, $h = 24\text{mm}$. A is chosen to maximize S_F.
$M = 5.574\text{mm}$
$L = 8.191\text{mm}$

	Calculation by presented model	Ray-tracing simulation
S_F $[\text{mm}^2]$	67.1	66.9
S_A $[\text{mm}^2]$	91.3	91.1
S_D $[\text{mm}^2]$	31.1	31.0
S_{TOTAL} $[\text{mm}^2]$	190	189

Conclusions

- We presented mathematical model of calculation of effective collecting area related not only to focal spot but also to focal cross arms and spots formed by direct rays.
- The results of the model are in good accordance to ray-tracing simulations.
- The model uses parameters of clear physical interpretation.
- For ideal mirror, collecting area Q_A related to arms of cross is equal to the area Q_F corresponding to focal spot. For real mirror, $Q_A > Q_F$.
- Therefore, image processing algorithms (e.g. deconvolution) that utilizes not the focal spot only but the full point-spread function can increase the effective collecting area more than twice.

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