LOPSIMUL - Quick Numerical Simulator of Multi-Foil Reflective Optical Systems

Vladimír Tichý

December 7, 2022

How the computation algorithm is made

Ray-tracing

- ▶ General usage
- ▶ Necessity of tracing of plenty of rays, therefore the calculation takes a long time.

Commonly, what is general, it is complex. Sometimes, the better way is to use simple model, simple equations, simple and fast code optimised for a given problem.

The aprogram is inteded for lobster eye and similar systems, e.g. Kirkpatrick-Baez that are composed of two orthogonal stack of mirrors.

Basic ideas how to optimize it for multi foil systems, particularly lobster eye:

- ▶ Separation of dimensions: 3D problem → two 2D problems that represents a small approximation
- ▶ Calculation of borders of reflections and shadows only that does not represent an approximation for flat mirrors.
- ▶ Usage of suited formalism makes the equation even simpler.

Separation of dimensions
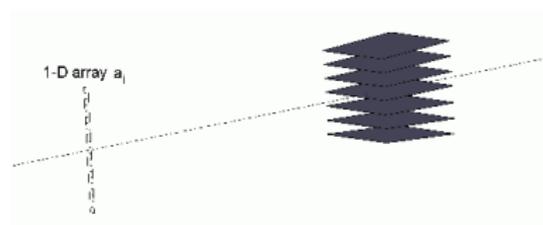
(a) First (horizontal) subsystem
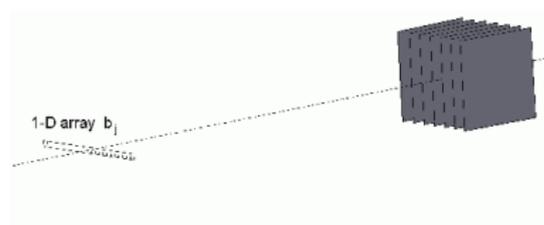
(b) Second (vertical) system
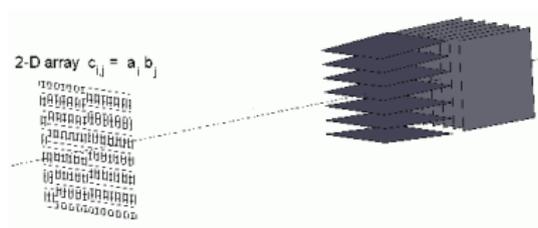
(c) Full 2D system

Figure: Separation of 2D system into two 1D systems

Look at one stack of mirrors

One stack of mirrors can be studied in the plane intersecting the system. In the optical system, some rays are reflected to the focus and other rays come through the system directly

FOCAL PLANE   MIRRORS
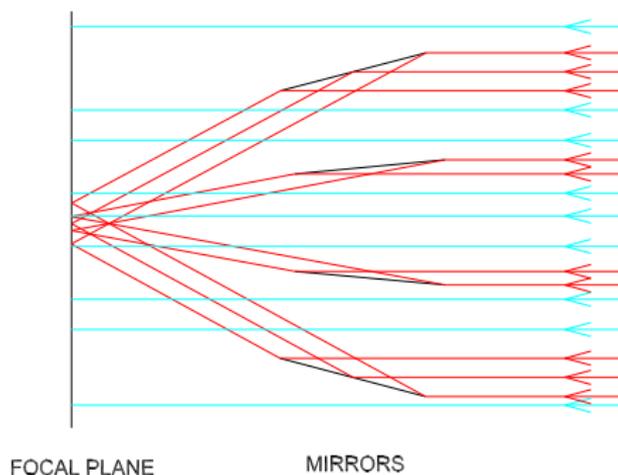
Used mathematical formalism

The fact, that the real two-dimensional space (plane) is isomorphic with the set of the complex numbers is utilized. So, the operations known of the complex numbers are used in an analogous manner to two-dimensional vectors. We got these operations:

Operation	Symbol	Definition
addition	$\mathbf{a} + \mathbf{b}$	$(a_1 + b_1, a_2 + b_2)$
subtraction	$\mathbf{a} - \mathbf{b}$	$(a_1 - b_1, a_2 - b_2)$
complex product	$\mathbf{ab}$	$(a_1 b_1 - a_2 b_2, a_1 b_2 + a_2 b_1)$
complex division	$\frac{\mathbf{a}}{\mathbf{b}}$	$\frac{1}{|\mathbf{b}|^2}(a_1 b_1 + a_2 b_2, a_2 b_1 - a_1 b_2)$
absolute value, norm	$|\mathbf{a}|$	$\sqrt{a_1{}^2 + a_2{}^2}$
complex conjugation	$\mathbf{a}^*$	$(a_1, -a_2)$
2-D vector inner product	$\mathbf{a} \cdot \mathbf{b}$	$a_1 b_1 + a_2 b_2$
2-D vector exterior product	$\mathbf{a} \times \mathbf{b}$	$a_1 b_2 - a_2 b_1$

Any other types of mathematical operations are not necessary.

Notation

- ▶ Common two-dimensional Cartesian coordinate system is used.
- ▶ Vectors are labeled by bold letters, e.g. $\mathbf{a}$.
- ▶ Coordinates are listed in round brackets and marked by subscript indices, e.g. (a_1, a_2).
- ▶ Common vector operations including the inner (dot) product and 2-D exterior (cross) products (whereof results are real numbers) are used.
- ▶ Indices that enumerate different vectors are written by superscript and marked by round brackets, e.g. $\mathbf{u}^{(1)}$, $\mathbf{u}^{(2)}$.

Direct rays

It is useful to begin the calculations by calculating the position of the shades of mirrors. The mirror can have non-zero thickness and its border is defined by points $\mathbf{M}^{(1)}$, $\mathbf{M}^{(2)}$, $\mathbf{M}^{(3)}$, $\mathbf{M}^{(4)}$.
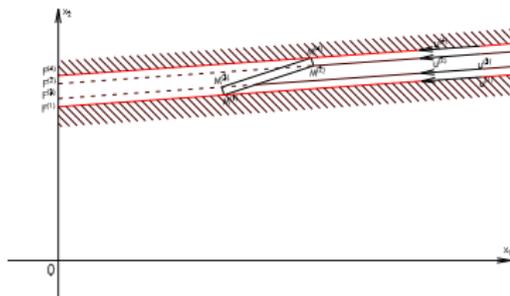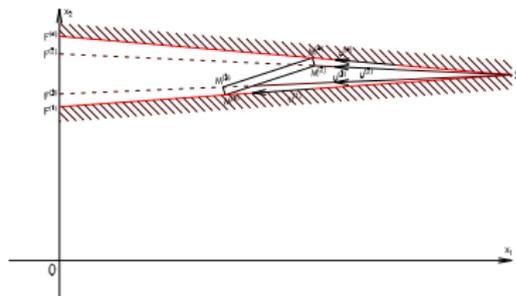
(a) Parallel beams   (b) Divergent beams

Equation for position of points $F^{(j)}$ is simple:

$$F_2^{(j)} = M_2^{(j)} - M_1^{(j)} \frac{u_2^{(j)}}{u_1^{(j)}} \tag{1}$$

The shadow lays between the minimal and maximal value of $F^{(j)}$, $j = 1 \ldots 4$.

Reflected rays
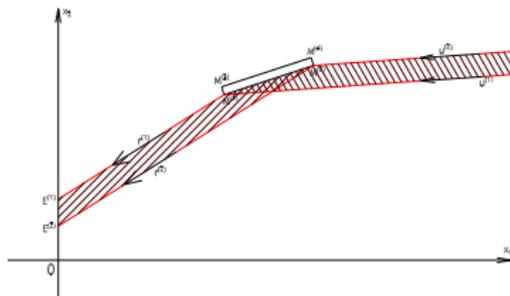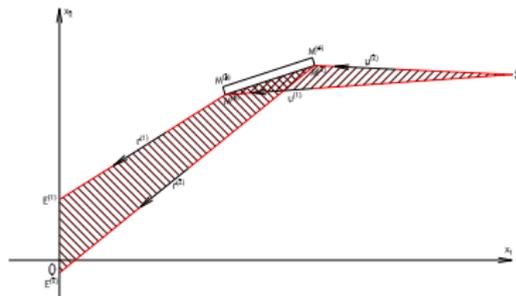
(c) Parallel beams   (d) Divergent beams

Direction of the reflected ray is calculated by equation

$$\mathbf{r}^{(j)} = \mathbf{m}\,\mathbf{m}\,\left(\mathbf{u}^{(j)}\right)^* \tag{2}$$

Positions of the borders of the reflection are calculated by equation

$$E_2^{(j)} = M_2^{(j+2s)} - M_1^{(j+2s)}\frac{r_2^{(j)}}{r_1^{(j)}} \tag{3}$$

Mirror direction vector $\mathbf{m}$ is defined as
$\mathbf{m} = \mathbf{M}^{(2)} - \mathbf{M}^{(1)}/|\mathbf{M}^{(2)} - \mathbf{M}^{(1)}|$ or
$\mathbf{m} = \mathbf{M}^{(4)} - \mathbf{M}^{(3)}/|\mathbf{M}^{(4)} - \mathbf{M}^{(3)}|$.

Shadows between mirrors

In dependence on the optics design and the incoming rays direction, some mirrors can be completely or partially shaded by adjacent mirror.
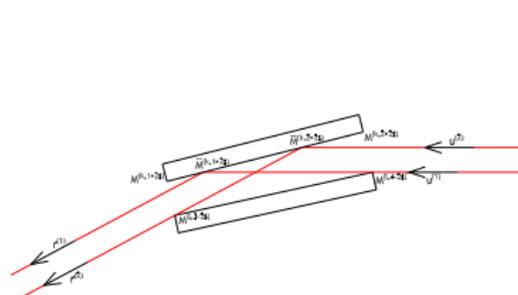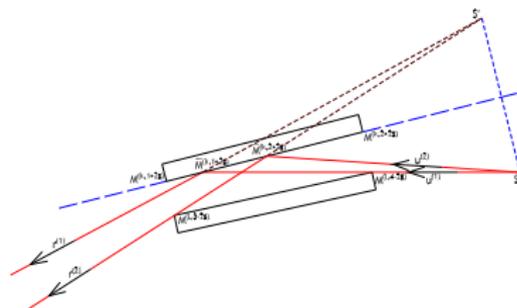
(e) Parallel beams   (f) Divergent beams

Figure: shadows between mirrors

In such case, in equations for calculations of positions of the reflections, vertices of the mirror have to be replaced by the points bordering the effective area. It is supposed that in the program, mirror data are arranged and index of the mirror on the reflecting side can be calculated e.g. $l = k + 2s - 1$,

Shadows between mirrors - parallel beams

The point $\overline{\mathbf{M}}^{(k,1+2s)}$ representing one of borders of active area of the mirror is given as an intersection of two lines $\mathbf{x}^{(1)}$, $\mathbf{x}^{(2)}$ of parametric equations

$$
\begin{array}{rcl}
\mathbf{x}^{(1)} & = & \mathbf{M}^{(k,1+2s)} + \alpha\mathbf{m} \qquad (4) \\
\mathbf{x}^{(2)} & = & \mathbf{M}^{(l,4-2s)} + \beta\mathbf{u}^{(1)} \qquad (5)
\end{array}
$$

Real parameters α, β defining the position of the intersection can by easily found as solution of a matrix system

$$
\left(\begin{array}{cc} m_1 & -u_1^{(1)} \\ m_2 & -u_2^{(1)} \end{array} \right) \left(\begin{array}{c} \alpha \\ \beta \end{array} \right) = \left(\begin{array}{c} M_1^{(l,4-2s)} - M_1^{(k,1+2s)} \\ M_2^{(l,4-2s)} - M_2^{(k,1+2s)} \end{array} \right) \qquad (6)
$$

Shadows between mirrors - parallel beams

Determinants of this system are

$$
\begin{aligned}
D &= \mathbf{u}^{(1)} \times \mathbf{m} \tag{7} \\
D_1 &= \mathbf{u}^{(1)} \times \left(\mathbf{M}^{(l,4-2s)} - \mathbf{M}^{(k,1+2s)} \right) \tag{8} \\
D_2 &= \mathbf{m} \times \left(\mathbf{M}^{(l,4-2s)} - \mathbf{M}^{(k,1+2s)} \right) \tag{9}
\end{aligned}
$$

Using the well known Cramer's rule, the solution of Eqs. (4)-(5) can be calculated as

$$
\alpha = \frac{D_1}{D}; \quad \beta = \frac{D_2}{D} \tag{10}
$$

$\beta < 0$ including the case $\beta = -\infty$ indicates reversed configuration.
$\alpha \geq 1$ (including the case $\alpha = +\infty$) indicates full shadows.
$\alpha \leq 0$ (including the case $\alpha = -\infty$) indicates no shadows.
If $0 < \alpha < 1$ then the mirror is partially shaded, point $\mathbf{M}^{(k,1+2s)}$ is replaced by point

$$
\overline{\mathbf{M}}^{(k,1+2s)} = \mathbf{M}^{(k,1+2s)} + \alpha\mathbf{m}. \tag{11}
$$

Shadows between mirrors - parallel beams

A similar method is used to calculate the point $\overline{\mathbf{M}}^{(k,2+2s)}$. The corresponding determinants have equations

$$D = \mathbf{r}^{(2)} \times \mathbf{m} \tag{12}$$

$$D_1 = \mathbf{r}^{(2)} \times \left(\mathbf{M}^{(l,3-2s)} - \mathbf{M}^{(k,1+2s)} \right) \tag{13}$$

$$D_2 = \mathbf{m} \times \left(\mathbf{M}^{(l,3-2s)} - \mathbf{M}^{(k,1+2s)} \right) \tag{14}$$

$$\alpha = \frac{D_1}{D}; \quad \beta = \frac{D_2}{D} \tag{15}$$

$\beta > 0$ indicates the opposite configuration. $\alpha \leq 0$ (including the case $\alpha = -\infty$) indicates full shadows. $\alpha \geq 1$ (including the case $\alpha = +\infty$) indicates no shadows. If $0 < \alpha < 1$ then the point $\mathbf{M}^{(k,2+2s)}$ is replaced by a point

$$\overline{\mathbf{M}}^{(k,2+2s)} = \mathbf{M}^{(k,1+2s)} + \alpha\mathbf{m}. \tag{16}$$

Shadows between mirrors - divergent beams
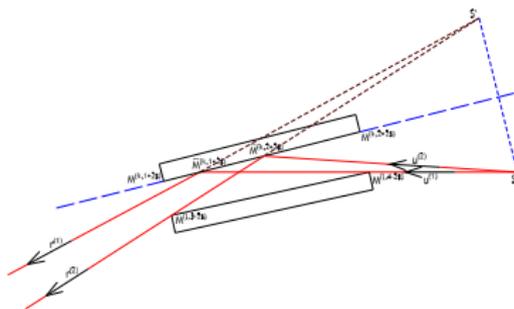
Vectors $\mathbf{u}^{(2)}$ and $\mathbf{r}^{(2)}$ are calculated in a different way, because the point $\overline{\mathbf{M}}^{(k,2+2s)}$ is not known at the beginning of the calculations. Outcoming ray of direction vector $\mathbf{r}^{(2)}$ comes from a virtual point $\mathbf{S}'$. The point $\mathbf{S}'$ lays in axially symmetric position of the point $\mathbf{S}$, where the symmetry axis is defined by the mirror surface.

$$\mathbf{S}' = \mathbf{M}^{(k,1+2s)} + \mathbf{mm}\left(\mathbf{S} - \mathbf{M}^{(k,1+2s)}\right) \qquad (17)$$

The vector $\mathbf{r}^{(2)}$ is calculated via equation

$$\mathbf{r}^{(2)} = \frac{\mathbf{M}^{(l,3-2s)} - \mathbf{S}'}{\left|\mathbf{M}^{(l,3-2s)} - \mathbf{S}'\right|} \qquad (18)$$

LOPSIMUL - Overview

LOPSIMUL - Control panel

Setup:
- optics arrangement, generation of lobster eye
- mirrors reflectivity
- source type (parallel, point, ...) and position
- simulation options
- simulation of manufacturing errors by generation of random deviations of mirrors positions

Movements of detector of 0.1mm, 1mm, 1cm, 1m

Generate demo optics for first usage by new users

File menu
(any mirror position is allowed if it is defined in the file)

Quick setup of main parameters

LOPSIMUL Control Panel

File   Setup   Detector move   Demo

Detector position

x 0   mm
y 0   mm
z 0   mm

Detector resolution

40   um

Optics rotation

yaw   0   deg
pitch 0   deg
axis   0   mm

Source position

x 0   mm
y 0   mm
z 57.2958 mm

Set
Retrieve
Center spot
Clear

RUN SIM

Use these parameters

Forgot these parameters

Center the spot (corner positions of reflections)

Clear the image

Run simulation (if "Auto run after data change" options is off)

LOPSIMUL - Image

Imaging options

Save image dialogue
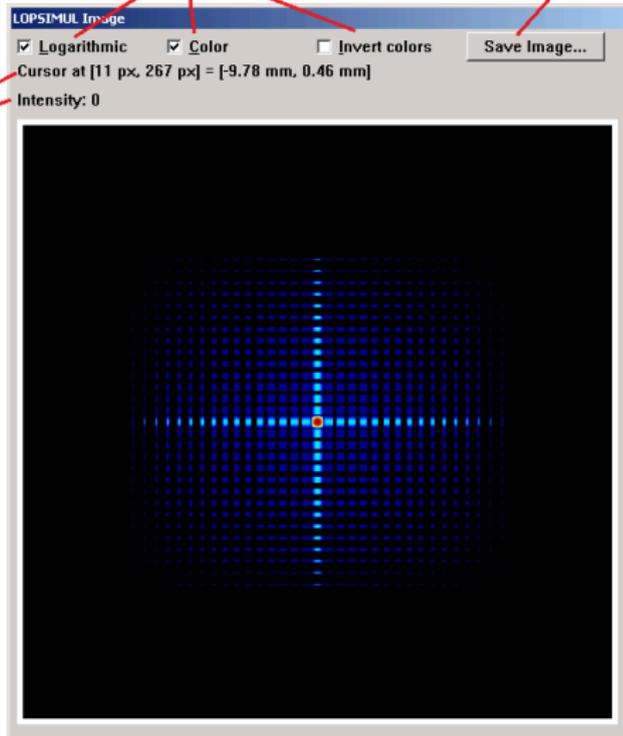
Currently, .bmp and .png formats are supported.

Data table as a text file that can be easily processed in other software is saved here, too.

When mouse cursor is moved over the image, detector and real coordinates of the corresponding point are shown together with intensity

LOPSIMUL - Graphs

Data can be filtered - intensity values can be replaced by average values of a strip of pixels of given size. If strip size is set to 1, this filtration is not applied.

Save dialogues (.bmp, .png and tables in a form of text files to be processed by other software)
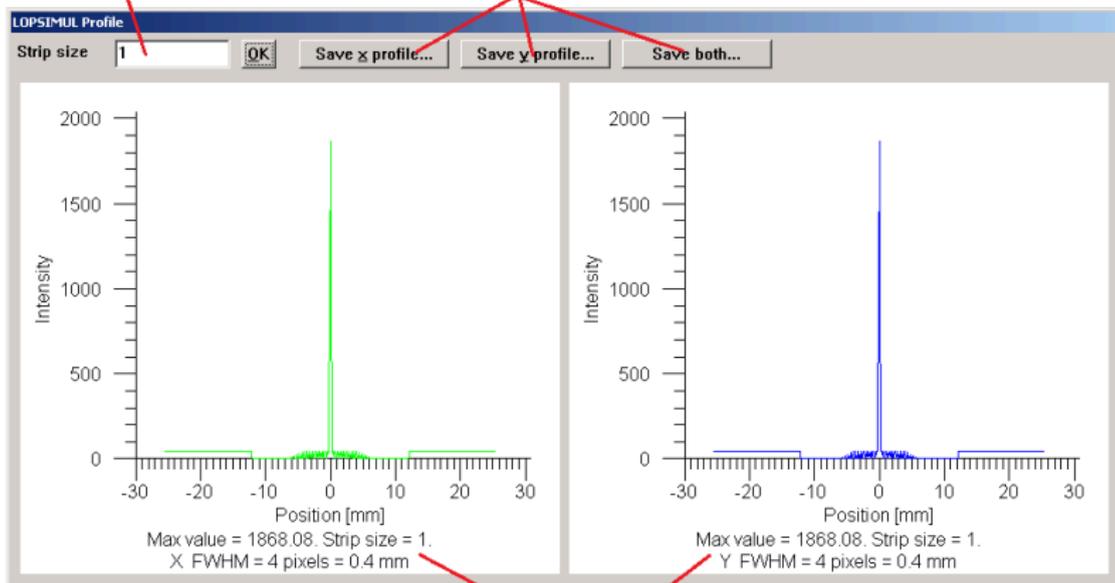
Maximal value (after the filtration) and FWHM are calculated and displayed.

LOPSIMUL - Results

Channel analysis gives

- ▶ Min refl, Max refl = border position of all reflections.
- ▶ MM effective collecting length = effective collecting length[2] related to the area bordered by Min rel and Max refl, i.e. by border rays that are reflected.
- ▶ MM width = width of focal image bordered by Min refl and max refl. It is not exactly equal to difference of Min refl and Max refl because of non-zero pixel size.
- ▶ FWHM = halfwidth of focal image.

LOPSIMUL - Results

The analysis of full 2-D image gives:

- ▶ MM spot integral = sum of intensities of all pixels laying within area bordered by Min refl and Max refl. I. e. it is total intensity in a rectangulare box bordered by all double-reflected rays.
- ▶ Peak position in pixel coordinates.
- ▶ Peak position in milimeters related to global coodinate system.
- ▶ Intesity of the peak.

For areas defined by FWHM and 10% rule, it is also calculated:

- ▶ Size of the area in pixels and square milimeters.
- ▶ Spot integral = sum of intensities of all pixels laying within this area.
- ▶ Gain

Where LOPSIMUL can be obtained?

- ▶ Downloaded at www.lopsimul.eu

How much does it cost?

- ▶ Usage of LOPSIMUL is free of charge.

There is an ask, anyway:

- ▶ If results obtained by LOPSIMUL are published anywhere (e.g. in article, paper, thesis, report, etc.), users are asked to mention there that this program was used and that the program uses simplified ray-tracing algorithm published in Exp. Astron. (2016) 41:377-392; DOI 10.1007/s10686-016-9493-2. Include citation of this paper and program homepage www.lopsimul.eu, please.

THANK YOU FOR THE ATTENTION